MMM	MMM	TTTTTTTTTTTTTT	ННН	HHH	RRRRRRRR	RRRR	TTTTTTTTTTTTTT	LLL
MMM	MMM	††††††††††††††††	ННН	ННН	RRRRRRRR		TTTTTTTTTTTTT	
MMM	MMM	ŤŤŤŤŤŤŤŤŤŤŤŤŤŤŤŤŤ	ННН	ннн	RRRRRRR		i i i i i i i i i i i i i i i i i i i	
MMMMMM	MMMMMM	111	нин	ннн	RRR	RRR	777	
MMMMMM	MMMMMM	+++						FFF
		111	HHH	ннн	RRR	RRR	ŢŢŢ	řřř
MMMMMM		!!!	ННН	HHH	RRR	RRR	ŢŢŢ	LLL
	MMM MMM	ŢŢŢ	HHH	HHH	RRR	RRR	TTT	LLL
	MMM MMM	111	HHH	HHH	RRR	RRR	TTT	LLL
MMM	MMM MMM	TTT	HHH	HHH	RRR	RRR	TTT	LLL
MMM	MMM	TTT	НИНИНИНИНИ		RRRRRRRR		ŤŤŤ	ĬĬĬ
MMM	MMM	TTT	НИНИНИНИНИ		RRRRRRRR		ŤŤŤ	<i>ו</i> ווֹ דּ
MMM	MMM	ŤŤŤ	НИНИНИНИНИ		RRRRRRRR		ŤŤŤ	iii
MMM	MMM	ŤŤŤ	ННН	ннн	RRR RR		ŤŤŤ	ili
MMM	MMM	ŤŤŤ	нин	ннн	RRR RR		ήii	
MMM	MMM	ή††	HHH	HHH	RRR RR		111	LLL
MMM		 T T						LLL
	MMM		ннн	ННН	RRR	RRR	ŢŢŢ	rrr
MMM	MMM	III	HHH	ННН	RRR	RRR	ŢŢŢ	LLL
MMM	MMM	TTT	ННН	HHH	RRR	RRR	TTT	LLL
MMM	MMM	TTT	ННН	HHH	RRR	RRR	TTT	
MMM	MMM	TTT	HHH	HHH	RRR	RRR	TTT	LLLLLLLLLLLLLL
MMM	MMM	111	ННН	HHH	RRR	RRR	ŤŤ	

MT MT MT MT MT

MT MT MT MT MT MT

• • • •

MM MM MMM MMM MMMM MMMM MMMM MM MM MM MM	TITITITIT TITITITIT TT TT TT TT TT TT TT	HH HHHHHHHHH	\$	RRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRR	
LL LL LL LL LL LL LL LL LL LL LL LL		\$			

; Floating Point Square Root routine MTH\$SQRT Table of contents 16-SEP-1984 01:51:08 VAX/VMS Macro V04-00 Page 0 55 83 123 205 HISTORY: Detailed Current Edit History
DECLARATIONS: Declarative Part of Module
MTH\$SQRT - Standard Single Precision Floating SQRT
MTH\$SQRT_R3 - JSB SQRT routine (2) (3) (4) (5)

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16-SEP-1984 01:51:08 6-SEP-1984 11:27:12 VAX/VMS Macro V04-00 [MTHRTL.SRC]MTHSQRT.MAR:1

Page (1)

```
0000
                    .TITLE MTH$SQRT
                                              Floating Point Square Root routine
0000
                                               (SQRT)
0000
                    .IDENT /1-015/
                                             : File: MTHSQRT.MAR
                                                                      EDIT RNH1015
0000
0000
0000
0000
               COPYRIGHT (c) 1978, 1980, 1982, 1984 BY
0000
               DIGITAL EQUIPMENT CORPORATION, MAYNARD, MASSACHUSETTS.
0000
               ALL RIGHTS RESERVED.
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          ; *
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; FACILITY: MATH LIBRARY

: ABSTRACT:

MTH\$SQRT is a function which returns the floating point square root of its single precision floating point argument. The call is standard call-by-reference. MTH\$SQRT_R3 is a special routine which is the same as MTH\$SQRT except a faster non-standard JSB call is used with the argument in RO and no

36 37 38 39 : registers are saved.

VERSION: 01

HISTORY: AUTHOR:

Peter Yuo, 15-Oct-76: Version 01

MODIFIED BY:

01-1 Peter Yuo, 22-May-77 01-2 Peter Yuo, 31-May-77

```
; Floating Point Square Root routine 16-SEP-1984 01:51:08 HISTORY; Detailed Current Edit History 6-SEP-1984 11:27:12
                                                                                                                    VAX/VMS Macro V04-00
                                                                                                                                                                        Page
                                                                                                                   [MTHRTL.SRC]MTHSQRT.MAR:1
         0000
                                             .SBTTL HISTORY; Detailed Current Edit History
                        556
557
558
50
         ŎŎŎŎ
         0000
                             : ALGORITHMIC DIFFERENCES FROM FP-11/C ROUTINE: none
         0000
         0000
                             ; Edit History for Version 01 of MTH$SQRT
         0000
                        61 62 63 64 65
         0000
                                            Code saying after code review
                                01-2 ROTL shift in garbage into highest bit. Use ASHL instead.

ADDL instruction after ADJUST has been changed into ADDW to prevent overflow if R1<31:16> = FFFF and R0<31:16> = FFFF
         0000
         0000
         0000
                       66
67
68
69
                                           Finish error handling 10-June-1977 MTH$$ERROR changed to MTH$$SIGNAL.
         0000
         0000
                                01-5
                                            MTH$_... changed to MTH__...
         0000
         0000
                                            Changed error handling mechanism. Put error result in RO before
                                calling MTH$$SIGNAL in order to allow user modify error result.

01-6 Return -0.0 on negative arg. TNH 20-Dec-77

01-7 Edit in Rich Lary's code bums. JSB routine is now R3. JMT 19-Jan-78

01-9 Move .ENTRY symbol to module header. TNH 14-Aug-78
         0000
                        70
71
72
73
74
75
         0000
         0000
         0000
                            : 1-010 - Put version number in standard format: three digit edit
numbers. Also, update the copyright notice. JBS 16-NOV-78
: 1-011 - Change MTH_SQUROONEG to MTH$K_SQUROONEG. JBS 07-DEC-78
: 1-012 - Add "" to the PSECT directive. JBS 22-DEC-78
: 1-013 - Declare externals. SBL 17-May-1979
         0000
         0000
                        76
77
78
79
         0000
         0000
         0000
                             1-014 - Move MTH$SQRT R2 to separate module (MTHSQRTR2.MAR) and replace with MTH$SQRT_R3. JAW 26-Sep-1979.
         0000
```

81 : 1-015 - Changed W^ to G^ in call to MTH\$\$SIGNAL RNH 09-Sept-1981

(2)

N 15

0000

0000

80

```
; Floating Point Squame Root routine 16-SEP-1984 01:51:08 VAX/VMS Macro V04-00 DECLARATIONS; Declarative Part of Modul 6-SEP-1984 11:27:12 [MTHRTL.SRC]MTHSQRT.MAR;1
            0000
0000
0000
                                     .SBTTL DECLARATIONS
                      ; Declarative Part of Module
                          INCLUDE FILES:
             0000
       EXTERNAL SYMBOLS:
                                     .DSABL GBL
                                     .EXTRN MTH$K_SQUROONEG
.EXTRN MTH$$51GNAL
                          EQUATED SYMBOLS:
0000400C
                                    ACMASK = ^M<IV, AS, R3>
                                                                     ; register save mask and IV enable
                     101
102
103
104
105
106
                             MACROS:
                                              none
                          : PSECT DECLARATIONS:
                                     .PSECT _MTH$CODE
                                                                   PIC, SHR, LONG, EXE, NOWRT
                                                                             ; program section for math routines
                     107
                     108
                             OWN STORAGE: none
                     109
                          : CONSTANTS:
                     112
113
114
115
            0000
                          ; Constants A and B chosen for k = odd
            0000
            ŎŎŎŎ
            0000
0000
0000
0000
0000
13CD5FD4
3C4A2018
                                    LF_ODD_A_E63
LF_ODD_B_EM63
                                                                   ^X13CD5FD4
                     116
                                                                   ^X3C4A2018
                     118
                             Constants A and B chosen for k = even
                     119
F61A4015
4B231FD7
                                    LF_EVEN_A = LF_EVEN_B_EM64 =
                                                                   ^XF61A4015
            ŎŎŎŎ
                                                                   ^X4B231FD7
```

3 (3)

```
MTH$SQRT
1-015
```

```
16-SEP-1984 01:51:08 VAX/VMS Macro V04-00
                                                                                                       Page
     MTH$SQRT - Standard Single Precision FL 6-SEP-1984 11:27:12 [MTHRTL.SRC]MTHSQRT.MAR;1
                  123
124
125
126
127
                               .SBITL MTH$SQRT - Standard Single Precision Floating SQRT
          ŎŎŎŎ
          ŎŎŎŎ
                      :++
: FUNCTIONAL DESCRIPTION:
          0000
          ŎŎŎŎ
          ŎŎŎŎ
          ŎŎŎŎ
                        SQRT - single precision floating point function
          ŎŎŎŎ
          ŎŎŎŎ
                  131
                        SQRT(X) is computed using the following approximation technique:
          ŎŎŎŎ
                  132
133
          ŎŎŎŎ
                               If X \le 0, error. Let X = \{X\}.
          ŏŏŏŏ
                  134
          ŎŎŎŎ
                  135
                               Let X = 2**K * F where F is the fractional part.
          0000
                  136
          0000
                  137
                               If K = \text{even}_{.} X = 2**(2P) * F
          0000
                  138
                                       SQRT(X) = 2**P * SQRT(F), 1/2 = < F < 1
          ŎŎŎŎ
                  139
          0000
                  140
                               If K = \text{odd}, X = 2**(2P+1) * F = 2**(2P+2) * (F/2)
          ČŎŎŎ
                  141
                                       SQRT(X) = 2**(P+1) * SQRT(F/2), 1/4 = < F/2 < 1/2.
          ŎŎŎŎ
                  142
          0000
                  143
                               Let F' = A*F + B,
          ŎŎŎŎ
                                                    = 0.453730314(octal),
                  144
                                                  B = 0.327226214(octal), for K = even.
          0000
                  145
                                      = A*(F/2) + B,
          0000
                  146
          0000
                  147
                                                  A = 0.650117146(octal)
          0000
                                                  B = 0.230170444(octal), for K = odd.
                  148
          0000
                  149
                               and
                                   K' = P,
= P + 1
          0000
                  150
                                                 for K = even
                                                 for K = odd.
          0000
                  151
          0000
                  152
                               Let YO = 2**K' * F' as a staight line approximation wthin the
          0000
                  153
          0000
                  154
                               given interval using coefficients A and B which minimize the
          0000
                  155
                               absolute error at the midpoint and endpoint.
          0000
                  156
          0000
                  157
                               Starting with YO, two Newton-Raphson iterations are performed.
          0000
                  158
          0000
                  159
                               Y[n+1] = (1/2) + (Y[n] + X/Y[n])
          0000
                  160
          0000
                  161
                               The relative error is < 10**-8.
          0000
                  162
          0000
                  163
                        CALLING SEQUENCE:
          0000
                  164
          0000
                  165
                               sqrt.wf.v = MTH$SQRT(x.rf.r)
          0000
                  166
          0000
                        INPUT PARAMETERS:
                  167
          0000
                  168
00000004
          0000
                  169
                                                                 ; define longword multiplier
                               LONG = 4
00000004
          0000
                  170
                               x = 1 + LONG
                                                                 ; Contents of x is the argument
          0000
                  171
                  172
173
          0000
                        IMPLICIT INPUTS:
                                                none
          0000
          0000
                  174
                        OUTPUT PARAMETERS:
                  175
          0000
          0000
                  176
                               VALUE: floating square root of the argument
                  177
          0000
          0000
                        IMPLICIT OUTPUTS:
                                                none
          0000
```

(4)

C 16

Floating Point Square Root routine

MTHSSORT 1-015	; Floating Point Square Root routing MTH\$SQRT - Standard Single Precision	16 e 16-SEP-1984 01:51:08 VAX/VMS on Fl 6-SEP-1984 11:27:12 [MTHRTL.	Macro V04-00 Page 5 SRC]MTHSQRT.MAR;1 (4)				
	0000 181: 0000 182: SIDE EFFECTS: 0000 183: 0000 184: Signals: MTH\$ SQUI 0000 185: the signal mechan 0000 186: Associated messag 0000 187: operand -0.0 unle 0000 188: 0000 189: NOTE: This proced 0000 190: overflow, causes 0000 191: preserves enables 0000 192: 0000 193:	Signals: MTH\$_SQUROONEG if X < 0.0 with reserved operand in RO (copied to the signal mechanism vector CHF\$L_MCH_RO/R1 by LIB\$SIGNAL). Associated message is: ''SQUARE ROOT OF NEGATIVE VALUE''. Result is reserved operand -0.0 unless a user supplied (or any) error handler changes CHF\$L_MCH_RO/R1 NOTE: This procedure disables floating point underflow, enables integer overflow, causes no floating overflow or other arithmetic traps, and preserves enables across the call.					
	0000 195 4000 0000 196 .ENTRY MTH 0002 197 0002 198 MTH\$FLAG_JA	SSORT, ACMASK ; standard call ; disable DV (a CKET ; flag that thi	-by-reference entry nd FU), enable IV s is a jacket procedure in				
6D 0000000°GF	9E 0002 MOVAB G^M 0009 0009	TH\$\$JACKET_HND, (FP) ; set handler a ; handler	ddress to jacket				
50 04 BC 01	0009 0009 199 50 0009 200 MOVF ax(10 000D 201 BSBB MTH 04 000F 202 RET 0010 203	; case of an er ; RO = arg \$SQRT_R3 ; call specail ; return - resu					

```
MTH$SQRT
1-015
```

```
: Floating Point Square Root routine
MTH$SQRT_R3 - JSB SQRT routine
                                                                     16-SEP-1984 01:51:08 VAX/VMS Macro V04-00 6-SEP-1984 11:27:12 [MTHRTL.SRC]MTHSQRT.MAR;1
                                                                                                                                Page
                                                                                                                                        (5)
                                   .SBTTL MTH$SQRT R3 - JSB SQRT routine
                           0010
                           0010
                                          JSB SQRT - used by the standard, and directly.
                           0010
                           0010
                                          CALLING SEQUENCE:
                                                 save anything in RO:R2
MOVF RO
JSB MTH$SQRT_R3
                           0010
                           0010
                                                                                       ; input in RO
                           0010
                           0010
                                                 return with result in RO
                           0010
                           0010
                                          Note: This routine is written to avoid any integer overflows, floating overflows,
                           0010
                                          floating underflows or divide by 0 conditions, whether enabled or not.
                           0010
                           0010
                                          REGISTERS USED:
                           0010
                                                 RO - Floating argument then result
                                   220
                           0010
                                                 R1 - X saved for use during iteration
                           0010
                                                 R2 - scratch
                           0010
                                       MTH$SQRT_R3:: MOVF
                           0010
                                                                                       ; JSB routine for SQRT ; test sign of X and save it in R1.
                50
53
                           0010
          51
                                                           RO, R1
                                                 BLEQ
                                                           ZERO_NEG
                                                                                       ; branch to ZERO_NEG if X =< 0
                           0015
0015
0015
0015
                                   x > 0
                                       POS:
                      3C
94
          52
                                                 MOVZWL
                                                          RO, R2
                                                                                         isolate low 16 bits (sign,exp,>fract) in R
                                                          R2
R2, R0
                           0018
                                                                                         R2 now has sign and left 7 exp bits
                                                 CLRB
                      AA 95
          50
                           001A
                                                 BICW
                                                                                         clear sign and left 7 exp bits
                50
                           001D
                                                 TSTB
                                                          RO
                                                                                         check low bit of exp
                10
                      18
                           001F
                                                 BGEQ
                                                           EVEN
                                                                                         and branch if 1
50
     13CD5FD4 8F
                      44
                           0021
                                                 MULF
                                                           #LF_ODD_A_E63, RO
                                                                                         add 64 (half of bias) to (exponent-2)
                                   236
237
238
                           0028
                                                                                         and start approximation calc
                      40
                                                                                         R0 = (first approx) * 2**-64
     3C4A2018 8F
                           0028
                                                 ADDF
                                                           #LF_ODD_B_EM63, RO
                           002F
                                                           ADJUST
                      11
                                                 BRB
                                                                                       ; qo adjust
                           0031
                                   239
                                   240 EVEN:
241
242
243
244 ADJUST:
245
246
247
248
                           0031
                                                          #^X2000, R0
#LF_EVEN_A, R0
          2000 8F
                           0031
    50
                                                 ADDW
                                                                                       : exp is 0 - make it 64 (2**-64) for legalit
     F61A4015 8F
                      44
                           0036
                                                 MULF
     4B231FD7 8F
                      40
                           003D
                                                 ADDF
                                                           #LF_EVEN_B_EM64, RO
                                                                                       ; R0 = (first approx) * 2**-64
                           0044
                      90
                           0044
    52
          52
                1F
                                                 ROTL
                                                          #31, R2, R2
                                                                                       ; divide R2 (exp+bias) by 2,
                           0048
                                                                                         giving (exp/2+64)
          50
                52
                      A0
                           0048
                                                 ADDW
                                                           R2, R0
                                                                                         insert exp/2 in first approx and
                           004B
                                                                                       ; re-bias it.
                           004B
                                        ; first iteration - single precision is sufficient
                           004B
                                   251
252
253
254
255
                           004B
          51 50
50 52
0080 8F
                                                          RO, R1, R2
R2, R0
    52
                      47
                           004B
                                                 DIVF3
                                                                                         R2 = X/YO
                      40
A2
                           004F
                                                 ADDF
                                                                                         RO = YO + X/YO
     50
                           0052
0057
                                                                                         R0 = Y1 = (1/2)(Y0 + X/Y0)
                                                          #^X80, RO
                                                 SUBW
                                                                                       ; no overflow possible
                                   256
257
258
259
260
261
                           0057
                           0057
                                          second iteration, do in double precision to get truncated( rather than
                           0057
                                          rounded) result.
                           0057
                                                                                       ; lower part (X) = 0
                           0057
                                                 CLRL
                                                          R2
R0, R1
                           0057
                                                 DIVD
                                                                                       ; divide Y1 into X with low-order
```

E 16

.END

MTH\$SQRT 1-015

```
G 16
MTH$SQRT
                                         : Floating Point Square Root routine
                                                                                            16-SEP-1984 01:51:08 VAX/VMS Macro V04-00
                                                                                                                                                           Page
Symbol table
                                                                                             6-SEP-1984 11:27:12 [MTHRTL.SRC]MTHSQRT.MAR:1
ACMASK
                   = 0000400C
ADJUST
                      00000044 R
EVEN
                                        01
                      00000031 R
LF_EVEN_A = F61A4015

LF_EVEN_B EM64 = 4B231FD7

LF_ODD_A_E63 = 13CD5FD4

LF_ODD_B_EM63 = 3C4A2018

= 00000004
                   = 0000004
LONG
MTH$SJACKET_HND
MTH$$SIGNAL
                                        Õ0
MTH$K_SQUROONEG
                                        00
                      ******
                      00000000 RG
MTH$SQRT
                                        01
                     00000010 RG
00000015 R
00000067 R
MTH$SQRT_R3
                                        Ŏ1
                                        01
POS
SQRTX
                                        01
                   = 00000004
ZERO_NEG
                      00000068 R
                                        01
                                                               Psect synopsis!
PSECT name
                                        Allocation
                                                                  PSECT No. Attributes
    ABS
                                        00000000 (
                                                           0.)
                                                                  00 ( 0.)
                                                                               NOPIC
                                                                                         USR
                                                                                                 CON
                                                                                                         ABS
                                                                                                                LCL NOSHR NOEXE NORD
                                                                                                                                            NOWRT NOVEC BYTE
                                                                 01 ( 1.)
 _MTH$CODE
                                        00000070 (
                                                                                  PIC
                                                                                         USR
                                                                                                 CON
                                                                                                        REL
                                                         124.)
                                                                                                                       SHR EXE
                                                                                                                                       RD
                                                                                                                                            NOWRT NOVEC LONG
                                                           Performance indicators !
Phase
                                Page faults
                                                  CPU Time
                                                                     Elapsed Time
Initialization
                                                   00:00:00.10
                                                                     00:00:00.81
Command processing
                                        109
                                                   00:00:00.72
                                                                     00:00:04.91
                                                   00:00:00.83
                                          82
                                                                     00:00:03.78
Pass 1
                                                                     00:00:00.00
                                           0
                                                   00:00:00.00
Symbol table sort
                                                  00:00:00.72
                                          64
Pass 2
Symbol table output
                                                                     00:00:00.41
Psect synopsis output
                                                   00:00:00.02
                                                                     00:00:00.02
Cross-reference output
                                                  00:00:00.00
                                                                     00:00:00.00
Assembler run totals
                                         291
                                                  00:00:02.42
                                                                     00:00:13.43
The working set limit was 900 pages.
4082 bytes (8 pages) of virtual memory were used to buffer the intermediate code.
There were 10 pages of symbol table space allocated to hold 17 non-local and 0 local symbols.
346 source lines were read in Pass 1, producing 11 object records in Pass 2.
1 page of virtual memory was used to define 1 macro.
                                                          Macro library statistics !
                                                         Macros defined
Macro library name
                                                                       0
 _$255$DUA28:[SYSLIB]STARLET.MLB;2
```

١

; Floating Point Square Root routine MTH\$SQRT VAX-11 Macro Run Statistics 16-SEP-1984 01:51:08 VAX/VMS Macro V04-00 6-SEP-1984 11:27:12 [MTHRTL.SRC]MTHSQRT.MAR;1 (5) O GETS were required to define O macros. There were no errors, warnings or information messages. MACRO/ENABLE=SUPPRESSION/DISABLE=(GLOBAL,TRACEBACK)/LIS=LIS\$:MTHSQRT/OBJ=OBJ\$:MTHSQRT MSRC\$:MTHJACKET/UPDATE=(ENH\$:MTHJACKET)+MSRC\$: 0263 AH-BT13A-SE

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